摘 要

本研究工作报告给出了单位圆盘 $\mathbb{D}$ 上由双倍权 ω 诱导的 Bergman 空间 A^p_ω 上的算子理论.

第一章介绍了双倍权 Bergman 空间 A^p_ω 及其算子理论的发展历程与研究背景, 并简要陈述了本研究工作获得的一些结果及其意义.

第二章研究了双倍权的一些特征，并给出了这些特征的一些应用. 第一个结果改进了已有的一个结论，证明了双倍权 ω 的若干几何特征的关键参数具有相同的下确界; 第二个结论表明，当 ω 是径向权且 $p>0$ 时，令 $\omega_{[p-2]}(z)=(1-|z|^2)^{p-2}\omega(z)$, 若 $\omega_{[p-2]}$ 是权函数, 则 $\omega_{[p-2]}$ 是双倍权当且仅当存在常数 σ 和 τ，对单位圆盘上任意解析函数 f(即 $f\in H(\mathbb{D})$), 有

$$\int_{\mathbb{D}}\int_{\mathbb{D}}\frac{|f(z)-f(\zeta)|^p}{|1-\overline{\zeta}z|^{4+\tau+\sigma}}\omega(\zeta)dA_\tau(z)A_\sigma(\zeta)\lesssim\int_{\mathbb{D}}|f'(z)|^p\omega(z)dA_{p-2}(z).$$

在此基础上, 研究了双倍权 Besov 函数的几何特征及相应空间上的 Hankel 型算子的有界性与紧性.

对任意 $f\in H(\mathbb{D})$, 令 $(If)(z)=\int_0^z f(\xi)d\xi$. 若 $0\le k<n$ 且 $g\in H(\mathbb{D})$, 广义 Volterra 积分算子 $T_g^{n,k}$ 定义为

$$T_g^{n,k}f=I^n(f^{(k)}g^{(n-k)}),\quad \forall f\in H(\mathbb{D}).$$

当 $n=1,k=0$ 时, $T_g^{n,k}$ 即为经典的 Volterra 积分算子. 若 B_z^ω 是 A^2_ω 的再生核, μ 是 $\mathbb{D}$ 上的正 Borel 测度, $k\in\mathbb{N}$, 广义 Toeplitz 算子 $\mathcal{T}^\omega_{\mu,k}$ 定义为

$$\mathcal{T}^\omega_{\mu,k}f(z)=\int_{\mathbb{D}}f^{(k)}(w)\overline{(B_z^\omega)^{(k)}(w)}d\mu(w),\ \ \forall f\in H(\mathbb{D}).$$

当 $k=0$ 时, $\mathcal{T}^\omega_{\mu,k}$ 即为经典 Toeplitz 算子 $\mathcal{T}^\omega_\mu$.

第三章研究了 ω 是双倍权时, 广义 Volterra 积分算子 $T_g^{n,k}:A^p_\omega\to A^q_\omega$ 的有界性和紧

性. 为了研究 A^2_ω 上的广义 Volterra 积分算子 $T^{n,k}_g$ 的 Schatten 类特征, 引入了 A^2_ω 上的广义 Toeplitz 算子 $\mathcal{T}^\omega_{\mu,k}$. 通过将 $\mathcal{T}^\omega_{\mu,k}: A^2_\omega \to A^2_\omega$ 分解为经典 Toeplitz 算子 $\mathcal{T}^\eta_\mu: A^2_\eta \to A^2_\eta$ 与若干其它算子, 得到了 $\mathcal{T}^\omega_{\mu,k}: A^2_\omega \to A^2_\omega$ 的有界性、紧性和 Schatten 类特征. 作为应用，得到了 $T^{n,k}_g: A^2_\omega \to A^2_\omega$ 的 Schatten 类特征. 该方法也被用于刻画 $T^{n,k}_g: H^2 \to H^2$ 的 Schatten 类特征.

第四章研究了 ω, η, υ 为满足一定条件的正规权时，Toeplitz 算子 $\mathcal{T}^\omega_\mu: A^p_\eta \to A^q_\upsilon (1 < p, q < \infty)$ 的有界性和紧性，并用一种更直接的方法，给出了 ω 是正规权且 $0 < p, q < \infty$ 时，A^p_ω 上的 q-Carleson 测度的一个新刻画.

关键词: 双倍权, Bergman 空间, 二重积分, Toeplitz 算子, Volterra 算子

Abstract

The aim of this report is to investigate the operator theory on Bergman spaces A^p_ω induced by doubling weights ω on the unit disk $\mathbb{D}$.

Chapter 1 is devoted to introduce the development and the background of the operator theory on Bergman spaces induced by doubling weights. Meanwhile, a brief introduction of our main results and the significance of them is given.

Chapter 2 is devoted to characterize the doubling weights and give some applications of these characterizations. The first one improves a known result, that is, the key parameters of some geometric characterizations of a doubling weight have the same infimum. The second one shows that, for a radial weight ω, letting $p > 0$ and $\omega_{[p-2]}(z) = (1 - |z|^2)^{p-2}\omega(z)$, if $\omega_{[p-2]}$ is a weight, $\omega_{[p-2]}$ is a doubling weight if and only if there exist real numbers σ and τ such that

$$\int_{\mathbb{D}}\int_{\mathbb{D}} \frac{|f(z)-f(\zeta)|^p}{|1-\overline{\zeta}z|^{4+\tau+\sigma}}\omega(\zeta)dA_\tau(z)A_\sigma(\zeta) \lesssim \int_{\mathbb{D}} |f'(z)|^p\omega(z)dA_{p-2}(z)$$

holds for all analytic functions f on $\mathbb{D}$(i.e. $f \in H(\mathbb{D})$). Based on these two results, we obtain some geometric characterizations of functions in Besov type spaces induced by doubling weights and investigate the boundedness and compactness of Hankel type operators related to these spaces

For any $f \in H(\mathbb{D})$, let $(If)(z) = \int_0^z f(\xi)d\xi$. When $0 \le k < n$ and $g \in H(\mathbb{D})$, the generalized Volterra integral operator $T_g^{n,k}$ is defined by

$$T_g^{n,k} f = I^n(f^{(k)}g^{(n-k)}),\ \ \forall f \in H(\mathbb{D}).$$

When $n = 1$ and $k = 0$, $T_g^{n,k}$ is the Volterra integral operator. Let B_z^ω be the reproducing kernel of A^2_ω, μ is a positive Borel measure on $\mathbb{D}$ and $k \in \mathbb{N}$, the generalized Toeplitz operator $\mathcal{T}^\omega_{\mu,k}$ is

defined by

$$\mathcal{T}_{\mu,k}^{\omega}f(z)=\int_{\mathbb{D}}f^{(k)}(w)\overline{(B_z^{\omega})^{(k)}(w)}d\mu(w),\ \forall f\in H(\mathbb{D}).$$

When $k=0$, $\mathcal{T}_{\mu,k}^{\omega}$ is the classical Toeplitz operator $\mathcal{T}_{\mu}^{\omega}$.

Chapter 3 is devoted to investigate the boundedness and compactness of generalized Volterra integral operators $T_g^{n,k}:A_\omega^p\to A_\omega^q$ when ω is a doubling weight. In order to describe the Schatten class of $T_g^{n,k}$ on A_ω^2, the generalized Toeplitz operator $\mathcal{T}_{\mu,k}^{\omega}$ on A_ω^2 is defined. By decomposing $\mathcal{T}_{\mu,k}^{\omega}:A_\omega^2\to A_\omega^2$ into a classical Toeplitz operator $\mathcal{T}_{\mu}^{\eta}:A_\eta^2\to A_\eta^2$ and some other operators, the boundedness, compactness and Schatten class of $\mathcal{T}_{\mu,k}^{\omega}:A_\omega^2\to A_\omega^2$ are characterized. As an application, the Schatten class of $T_g^{n,k}$ on A_ω^2 is characterized. This method is also used to characterize the Schatten class of $T_g^{n,k}:H^2\to H^2$.

In Chapter 4, the boundedness and compactness of $\mathcal{T}_{\mu}^{\omega}:A_\eta^p\to A_\upsilon^q$ with some regular weights ω,η,υ and $1<p,q<\infty$ are studied. Then, a direct way is used to obtain a new characterization of q-Carleson measures for A_ω^p when ω is regular and $0<p,q<\infty$.

Key Words: doubling weighs, Bergman spaces, double integral, Toeplitz operator, Volterra operator